MOLECULAR PHYSICS REPORTS **38** (2003) 74-78

FAST MAGNON RELAXATION

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Abstract. Magnon relaxation times are usually calculated by solving linearized Boltzmann-type equation for magnon occupation numbers. If in the initial state the occupation number of a particular magnon mode is high, non-linear terms in the Boltzmann equation strongly enhance the short-time relaxation rate for that magnon mode. The problem will be discussed in details for a case of three-magnon confluence processes for degenerate magnon spectrum. The same qualitative features characterize higher order relaxation processes like the four-magnon ones.

1. INTRODUCTION

To begin with, what we mean by “fast” relaxation? It will be shown below that the initial relaxation rate of magnons after abrupt strong disturbance from an equilibrium state is enhanced by a contribution due to non-linear effects. This more rapid, early stage of magnon relaxation will be referred here as “fast relaxation”. (It should not be confused, in particular, with the special relaxation mechanism acting in some materials, see e.g. [1]).

Relaxation processes in ferromagnets were studied intensively in 50-ties and 60-ties of the last century. Fundamentals are well established and summarized in excellent monographs by Keffer [2], Sparks [1] and others. Studies of various relaxation processes were motivated by efforts to explain the observed line width ΔH of the ferromagnetic resonance (FMR). In the model ferromagnetic material which is the iron-yttrium garnet (YIG) theory of relaxation is very successful (see [1, 2]), not quite so in many other ferromagnetic materials in which a plethora of mechanisms contribute to damping of the resonance line. In ferromagnetic resonance the line width ΔH is often proportional to a sum of relaxation rates (or inverse relaxation times) of a few relaxations mechanism which dominate in a particular material and given conditions. Precession of magnetization in ferromagnetic resonance can be considered as excitations of magnons driven by the applied microwave field, magnons then decaying by relaxation mechanisms.

Magnon relaxation times are calculated for small disturbances of an equilibrium state by linearizing Boltzmann type equation for time evolution of magnon occupation numbers or, equivalently by the Kubo linear response theory.

Non-linear effects in ferromagnetic resonance at high microwave power were studied already 4-5 decades ago (see [3]), experiments on parallel pumping provided a method of accurate measurements of magnon relaxation time in materials like YIG [1]. Later on, non-linear behaviour of magnon excited by high power microwaves were studied looking for chaotic phenomena in magnon systems (see e.g. [4]).

In the last decade or so powerful techniques were developed for studying dynamics of magnetization in ferromagnets following strong pulses of magnetic field in an amazingly fine time scale, in the femtoseconds range (see e.g. [5-9]).

These pulse experiments are the motivation for the present discussion of a simple magnon relaxation mechanism which leads to an interesting anomaly in time evolution of magnon occupancies.

2. THREE-MAGNON CONFLUENCE PROCESSES

We shall consider here the simplest relaxation processes in which nonlinear effects can play a role. These are the 3-magnon confluence processes due to dipolar interaction of magnetic moments of electrons responsible for ferromagnetism (see e.g. [2] for the background information). In numerical estimates the material parameters of YIG will be taken, however the following considerations are in fact model independent and can be applied to itinerant electron ferromagnets as well (in this connection see [10]).

The 3-magnons confluence processes are the ones in which two magnons, of wave vectors say $\vec{k}$ and $\vec{q}$, annihilate and another magnon of momentum $(\vec{k}+\vec{q})$ is created. The transition rate or the number of transitions in a unit of time is $W_{kq} n_k n_q (n_{k+q}+1)$ where n_k is the number of magnons $\vec{k}$ and W_{kq} is the transition probability calculated from the Fermi golden rule,

$$W_{kq} = \frac{1}{\tau} N^{-1} \,|\, f_k + f_q |^2 \, \delta(\varepsilon_k + \varepsilon_q - \varepsilon_{k+q}) \tag{1}$$

The Dirac δ-function takes care of the magnon energy conservation and, for dipolar forces $f_k = k_z(k_x + ik_y)/k^2$ and $1/\tau \sim (\mu_B M)^2/(\hbar k_B)$, ($\tau \cong 1.7 \times 10^{-12}$ sec. for YIG), M is the magnetization. The rate of change dn_k/dt of the population of magnons $\vec{k}$ is a balance between the rate of their decaying and the inverse processes at the rate $W_{kq}(n_k+1)(n_q+1)n_{k+q}$,

$$\frac{dn_k}{dt} = -\sum_q W_{kq} n_k n_q (n_{k+q}+1) + \sum_q W_{kq} (n_k+1)(n_q+1) n_{k+q} \tag{2}$$

(cf. [2]). The magnon energy in the presence of dipolar interactions, for wave vectors k large as compared with inverse dimensions of a sample and small as compared with inverse lattice spacing, is

$$\varepsilon_k = \left\{ \left(2\mu_B H + Dk^2\right)\left(2\mu_B H + 8\pi\mu_B M \sin^2\vartheta_k + Dk^2\right)\right\}^{1/2} \tag{3}$$

where D is the magnon stiffness constant, H is the internal magnetic field (including the demagnetization term, $H = H_{\mathrm{ext}} - 4\pi N_z M$) and $\sin^2\vartheta_k = (k_x^2 + k_y^2)/k^2$. For practical calculations, the degenerate spectrum (3) is customary approximated by simpler expression $\varepsilon_k = 2\mu_B H + Dk^2 + 4\pi\mu_B M \sin^2\vartheta_q$ with a slight loss of accuracy. If the magnon occupation number $n_k(t)$ is close to its equilibrium value at a given temperature $n_k^0 = 1/(e^{\varepsilon_k/k_B T} - 1)$, i.e. if $\zeta_k(t) = n_k(t) - n_k^0$ is small then the Boltzmann equation (2) can be linearized with respect to the deviations ζ_k and (2) can be approximated by

$$\frac{d\zeta_k}{dt} = -\frac{1}{\tau_k}\zeta_k + \dots \tag{4}$$

where

$$\frac{1}{\tau_k} = \sum_q W_{kq} (n_q^0 - n_{k+q}^0)\,. \tag{5}$$

τ_k ($\equiv \tau_k^{(2,1)}$) is the 3-magnon confluence relaxation time and its meaning is clear from the obvious solution of Eq. (4), $\zeta_k(t) = \zeta_k(0)\, e^{-t/\tau_k}$. The above procedure illustrated by the simple example of the 3-magnon confluence processes is standard for discussions of relaxation processes (see [2]).

Now we try to solve the Boltzmann equation (2) for arbitrary deviations $\zeta_k(t)$ from equilibrium, possibly large. Formally one can write Eq. (2) as

$$\frac{d\zeta_k(t)}{dt} = -\left\{\frac{1}{\tau_k} + \psi_k(t)\right\}\zeta_k(t) + \psi_{1k}(t) \tag{6}$$

where

$$\psi_k(t) = \sum_q (W_{kq} - W_{k,q-k})\zeta_q(t) \tag{6a}$$

and

$$\psi_{1k}(t) = -n_k^0\psi_k(t) + \sum_q W_{kq}\left[\left(n_q^0 + 1\right)\zeta_{q+k} + n_{q+k}^0\zeta_q\right] + \sum_q W_{kq}\zeta_q\zeta_{q+k} \tag{6b}$$

are not neglected now. The formal solution of (6), if ψ_k and ψ_{1k} were known, is

$$\zeta_k(t) = \left\{\zeta_k(0) + \alpha_k(t)\right\}e^{-t/\tau_k - \eta_k(t)} \tag{7}$$

where

$$\eta_k(t) = \int_0^t du\,\psi_k(u)\,, \tag{7a}$$

$$\alpha_k(t) = \int_0^t du\,\psi_{1k}(u)e^{u/\tau_k + \eta_k(u)}\,. \tag{7b}$$

As an initial condition we assume that at $t = 0$ magnons of one particular wave vector $\vec{Q}$ are strongly excited, i.e. $\zeta_q(0) = f\delta_{k,Q}$ where f is a large number. For $f \gg 1$, retaining only the leading term $\sim f$ we have from (7a) (for $\vec{k} = \vec{Q}$):

$$\eta_Q(t) \cong fW_{QQ}\int_0^t du\,e^{-u/\tau_Q - \eta_Q(t)}\,. \tag{8}$$

The equation (8), a self-consistency condition, can be solved exactly to give

$$\eta_Q(t) = \ln\left[1 + \tau_Q\, fW_{QQ}\left(1 - e^{-t/\tau_Q}\right)\right]. \tag{9}$$

In the limit $t \ll \tau_Q$, the approximation $\eta_Q(t) \cong fW_{QQ}t$ holds and (if $f \gg 1$) from (7) it follows:

$$\zeta_Q(t) \cong fe^{-(1/\tau_Q + fW_{QQ})t}\,. \tag{10}$$

Therefore, for $t \ll \tau_Q$ the effective relaxation frequency of magnons $\vec{Q}$,

$$\frac{1}{\tau_Q^f} \stackrel{\text{def}}{\equiv} \frac{1}{\tau_Q} + fW_{QQ} \tag{11}$$

is enhanced by the contribution fW_{QQ} (> 0). Exactly this is what we mean by "fast relaxation": in the "initial" time interval, from $t = 0$ to $t \ll \tau_Q$ the effective relaxation time τ_Q^f is shorter than τ_Q, calculated from linearized Boltzmann Eq. (4). It is remarkable that the difference $1/\tau_Q^f - 1/\tau_Q$

is proportional to the initial disturbance i.e. the number f of magnons Q initially excited and is temperature independent. Both properties can be easily understood: if at $t = 0$ a large number f of magnons $\vec{Q}$ is created, by far outweighting all other thermally excited magnons in the system, the dominant relaxation processes consist of decaying two magnons $\vec{Q}$ with creating a magnon of wave vector $(2\vec{Q})$. As t increases beyond τ_Q, $\eta_Q(t)$ approaches a constant value and the time evolution of $\zeta_Q(t)$ is determined by the standard relaxation time τ_Q.

3. HIGHER ORDER PROCESSES

The conclusion (11) that initial relaxation frequency is enhanced over the one resulting from linearized Boltzmann equation is valid also for higher order magnon relaxation processes. The magnon Hamiltonian for a ferromagnet (either of localized or itinerant electrons) with magnetic dipolar interactions taken into account contains 3- and 4-magnon interactions. Besides the 3-magnon (2,1) confluence processes discussed in details in Section 2 there are 4-magnon (2,2) scattering processes (a given magnon $\vec{k}$ collides with another one $\vec{k}'$ giving magnons $\vec{k} + \vec{q}$ and $\vec{k}' - \vec{q}$) due to both exchange and dipolar interactions and (3,1) confluence processes (a magnon $\vec{k}$ and two other, $\vec{q}$ and $\vec{q}'$ vanish to produce a magnon $\vec{k} + \vec{q} + \vec{q}'$). The so-called splitting processes, 3-magnon ((1,2) i.e. $\vec{k} = \vec{q} + (\vec{k} - \vec{q})$) or 4-magnon ((1,3) i.e. $\vec{k} = \vec{q} + \vec{q}' + (\vec{k} - \vec{q} - \vec{q}')$) are not considered here since they are allowed by energy conservation conditions only if magnons wave vectors k exceed a threshold value $k_{\min}$. Prospects of a possible experimental verification of predictions for large $k > k_{\min}$ seem to be remote, so they are not discussed here.

The time evolution of $\zeta_k(t) = n_k(t) - n_k^0$ in the presence of higher order magnon scattering processes has the same form as (6) but now $1/\tau_k = 1/\tau_k^{(2,1)} + 1/\tau_k^{(2,2)} + 1/\tau_k^{(3,1)}$ and

$$\psi_k(t) = \sum_q A_q^k \zeta_q + \sum_{qq'} B_{qq'}^k \zeta_q \zeta_{q'}. \tag{12}$$

In the limit $f \gg 1$, the explicit expression for $\psi_{1k}(t)$ is not relevant so we do not quote it. $\tau_k^{(2,1)}$, $\tau_k^{(2,2)}$ and $\tau_k^{(3,1)}$ are the relaxation times calculated from linearized Boltzmann equation for 3-magnon confluence processes (2,1) (given by Eq. (5)), 4-magnon (2,2) scattering processes and 4-magnon (3,1) confluence processe, respectively. Also, the coefficients A_q^k and $B_{qq'}^k$ contain contributions from these interactions, $A_q^k = A_q^{(2,1)k} + A_q^{(2,2)k} + A_q^{(3,1)k}$ where e.g. $A_q^{(2,1)k} = W_{kq} - W_{k,q-k}$, (cf. Eq. (6a)). Explicit expressions for the coefficients will be published elsewhere, with a little effort they can be reproduced from formulae given in the reference [2].

As previously we assume that at time $t = 0$ a large number of particular magnons $\vec{Q}$ is excited, i.e. $\zeta_k(t) = f\delta_{k,Q}$, $f \gg 1$. The same arguments previously explained, Eqs. (7-11), lead to the conclusion that in an initial interval of time, $t \ll \tau_Q$ the effective relaxation frequency $1/\tau_Q^f$ is again enhanced over $1/\tau_Q$ by a contribution $f A_Q^Q$. However, it appears that B_{QQ}^Q can have a non-vanishing value for the 4-magnon (3,1) confluence processes of dipolar origin. Therefore a term $f^2 B_{QQ}^Q$ is also present in $1/\tau_Q^f$,

$$\frac{1}{\tau_Q^f} = \frac{1}{\tau_Q} + f A_Q^Q + f^2 B_{QQ}^Q \tag{13}$$

Although B_{QQ}^{Q} is much smaller than A_{Q}^{Q}, for large f both corrections in (13) can reach comparable values. Again, the corrections to the effective short-time relaxation frequency $1/\tau_Q^f$ are 1° temperature independent and 2° increase with the strength of the disturbance f. These two features could be the basis for a possible experimental verifications, perhaps in pulse experiments.

4. DISCUSSION

The temperature independent correction to the relaxation rate fW_{QQ}, Eq. (11), depends on Q and on the angle ϑ_Q between $\vec{Q}$ and a direction of an applied magnetic field. A simple estimate of the order of magnitude of the correction can be given considering fW_{QQ} averaged over direction of $\vec{Q}$:

$$f\langle W_{QQ}\rangle = fN^{-1}\frac{1}{\tau}2\left(\frac{DQ^2}{2\pi\mu_B M}-\frac{H}{2M}\right)\left(1+\frac{H}{2M}-\frac{DQ^2}{2\pi\mu_B M}\right)^{1/2} \tag{14}$$

if Q satisfies the condition

$$\frac{H}{2M} < \frac{DQ^2}{2\pi\mu_B M} < 1 + \frac{H}{2M} \tag{15}$$

The conditions (15) can be satisfied for magnons Q in the degenerate spectrum (e.g. for YIG, at $H \sim 3$kOe for $Q \sim 0.02$ in units of the inverse lattice constant). Thus $f\langle W_{QQ}\rangle$ is equal to $fN^{-1}\ 1/\tau$ multiplied by the function of Q varying between 0 and its maximal value $4 \times 3^{-3/2}$. Therefore the contribution $f\langle W_{QQ}\rangle$ to the 3-magnon confluence relaxation rate is of the order of magnitude $(f/N)\,1/\tau$. For YIG $f\langle W_{QQ}\rangle \sim 5 \times 10^{11}$ (f/N) 1/sec so even for small (f/N) the temperature independent contribution fW_{QQ} to $1/\tau_Q^f$, Eq. (11), can be comparable with $1/\tau_Q$. For YIG $1/\tau_Q$ is a linear function of temperature, for $Q \cong 0.02$ given by $1/\tau_Q \cong 2 \times 10^6\ (T/300^\circ\mathrm{K})$ 1/sec (cf [1]).

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